WADC TECHNICAL REPORT 53-286

6559

AD0021909 6559

VIBRATIONS OF A HELICOPTER ROTOR SYSTEM AND FUSELAGE INDUCED BY THE MAIN ROTOR BLADES IN FLIGHT

One of a Series of Reports on Helicopter Vibration

MORRIS MORDUCHOW
SHAO W. YUAN
K. E. PERESS

POLYTECHNIC INSTITUTE OF BROOKLYN

20030818053

JUNE 1953

WRIGHT AIR DEVELOPMENT CENTER

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

The information furnished herewith is made available for study upon the understanding that the Government's proprietary interests in and relating thereto shall not be impaired. It is desired that the Judge Advocate (WCJ), Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, be promptly notified of any apparent conflict between the Government's proprietary interests and those of others.

McGregor & Werner, Inc., Wakefield, Mass.
Nov. 24, 1953 100

Errata to AD 21969

WADC TECHNICAL REPORT 53-286 June 1953

ERRATA - January 1954

The following is a corrected list of authors for WADC Technical Report 53-286, "Vibrations of a Helicopter Rotor System and Fuselage Induced by the Main Rotor Blades in Flight," dated June 1953:

Morris Morduchow
Shao W. Yuan
H. Reissner

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

AF-WP-(B)-O-11 FEB 54 100

WADC TECHNICAL REPORT 53-286

VIBRATIONS OF A HELICOPTER ROTOR SYSTEM AND FUSELAGE INDUCED BY THE MAIN ROTOR BLADES IN FLIGHT

One of a Series of Reports on Helicopter Vibration

Morris Morduchow
Shao W. Yuan
K. E. Peress

Polytechnic Institute of Brooklyn

June 1953

Aircraft Laboratory
Contract No. AF 18(600)-154
RDO No. 455-45

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

FOREWORD

This report was prepared at the Polytechnic Institute of Brooklyn, and was sponsored by the Aircraft Laboratory, Directorate of Laboratories, Wright Air Development Center, under Contract No. AF 18 (600)-154, Research and Development Order No. 455-45, "Helicopter and Convertiplane Vibration," with Dr. Orville R. Rogers acting as project engineer. This report is the second of two technical reports submitted under this contract. The first report, WADC TR 53-355, "Helicopter Blade-Forces Transmitted to the Rotor Hub in Flight," was published simultaneously.

ABSTRACT

Based on a simplified model of the hub-fuselage structure, a theoretical analysis is made of the response of the hub and fuselage of a helicopter in flight to harmonic forces transmitted by the rotor blades to the hub both in, and normal to, the plane of rotation. The assumed structure is in the form of a plane framework with masses concentrated at the joints. Simple expressions are derived for the vibration amplitudes of the mass points as functions of the masses and natural frequencies of the hub and the fuselage. The pertinent non-dimensional parameters are determined, and simple explicit conditions of resonance are derived. Numerical examples are given to illustrate the results.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

H. P. Lippman
for DANIEL D. MCKEE
Colonel, USAF
Chief, Aircraft Laboratory
Directorate of Laboratories

TABLE OF CONTENTS

SYMBOLS

E	modulus of elasticity of material of a bar.
e_{ix}, e_{iz}	amplitudes of elastic deflections of mass point i (i=1, 2, 3) in x and z directions, respectively, due to exciting force in rotor plane of rotation (F_1=0).
e_{ixr}, e_{izr}	amplitudes of rigid-body deflections of mass point i in x and z directions, respectively, for F_1=0.
e_{ixt}, e_{izt}	amplitudes of total deflections of mass point i in x and z directions, respectively, for F_1=0.
e_{ix}', e_{iz}', e_{ixr}' etc.	same as corresponding unprimed quantities, except that exciting force is normal to plane of rotation (F_2=0).
F_z, F_x	exciting harmonic forces in z and x directions, respectively.
F_1, F_2	amplitudes of F_z and F_x, respectively (eqs. (2a) and (2b)).
f	$= \omega_f/\omega_h = \varphi/H$. Ratio of fuselage natural frequency to hub natural frequency.
H	$= \omega_h/\Omega$. Ratio of hub natural frequency to rotor angular speed.
$H_{res.}$	value of H for resonance.
k_A, k_B, k_C	spring moduli of bars A, B and C respectively. (Fig. 1). $k_B=k_A$.
l	length of a bar.
M	$= m_1/m_2$.
m_1	total mass of blades, pylon and blade-hub connections.
m_2	half of the gross mass of the fuselage.

n	number of rotor blades.
S	cross-sectional area of a bar.
t	time
u_i, v_i	(time-dependent) total deflections of mass point i (i=1, 2, 3) in the x and z directions respectively. (Fig. 1).
x	direction of intersection of rotor plane of rotation with plane of simplified hub-fuselage structure (Fig. 1).
z	direction normal to plane of rotation
ν	ratio of frequency of exciting harmonic force to rotor angular speed. (eqs. (2a) and (2b)).
θ	angle in hub-fuselage structure (Fig. 1).
Ω	rotor angular speed
ω_f	natural frequency of fuselage (see also eq. (8)).
ω_h	natural frequency of hub structure (see also eq. (7)).
φ	$= \omega_f/\Omega$.
$\varphi_{res.}$	value of φ for resonance.

INTRODUCTION

This report represents the second phase of research sponsored by the Air Force on the response of various structural parts of a helicopter to the harmonic forces transmitted in flight by the rotor blades to the hub. In the first phase of this research, a detailed analytic determination (reference 1) was made of the forces produced by the helicopter blades at the hub. The connections between the hub, pylon and its support on the fuselage were assumed as rigid, so that the hub and pylon would have only a negligible influence on the blade forces. In the present report, the forces transmitted to the hub are considered as known, and the response of the helicopter hub and fuselage to these forces is investigated. The problem here is thus essentially one of the forced vibrations of an elastic structure induced by given external exciting forces.

Although considerable work on ground vibrations of helicopters has been accomplished (for example, references 2 and 3), relatively little theoretical research has been performed on the response of rotors to harmonic exciting forces in flight. The chief work, which is fairly recent, along these lines is reference 4, where an analysis is made of the response of helicopter rotors to harmonic blade-forces in the plane of rotation. The method of analysis there is based on the ground vibration analysis of reference 2.

Since the actual helicopter rotor structure is complicated, it is necessary, in analyzing theoretically the response of a rotor to harmonic forces, to make certain simplifying assumptions. In particular it is necessary to assume a simplified rotor and fuselage, but nevertheless to retain the essentials of the actual structure. For this purpose, a particularly simple physical model of the hub-fuselage structure, in the form of a plane framework with masses concentrated at the joints, is assumed in the present analysis. In spite of the simplicity of such a model, it will be seen that it includes most of the essentials of the actual helicopter configuration, such as the mass and elastic characteristics of both the hub and the fuselage.

The analysis in this investigation is in a sense an extension of that in reference 4, since the present investigation includes exciting harmonic forces and consequent motions not only in the rotor plane of rotation, but also normal to the plane. In fact, the rotor response to forces normal to the plane will be shown here to be a function of the natural frequency of the fuselage, although the response to exciting forces in the plane of rotation is not a function of this property. Consequently, the natural frequency of the fuselage does not appear as a parameter in reference 4.

Since the structural model analyzed here is considerably different from that treated in reference 4, the method of analysis is also different. In reference 4, the Lagrange equations of motion are applied to a system of (n+2) degrees of freedom (n= number of rotor blades), while in the present investigation, a fairly simple application is made of D'Alembert's principle to the motion of three mass points essentially connected by springs. In spite of such differences the final results derived here are, for the rotor response to an exciting force in the plane of rotation, quite similar to those of reference 4.

I

The Simplified Structure

The simplified hub-fuselage structure assumed in the present analysis is shown in Figure 1. This may be regarded as a symmetrical plane framework with masses concentrated at the joints. The bars connecting the three joints may be assumed as free of bending vibration, so that only longitudinal vibrations occur in the bars, i.e. the bars act only in tension or compression. The mass m_1 at joint 1 consists primarily of the total mass of the rotor blades and of the pylon-hub-blade connections. The masses m_2 at joints 2 and 3 may each be considered as equal to half of the gross mass of the fuselage. As in the case of a helicopter in flight, the structure in Figure 1 is assumed to be suspended freely in space by the lift on the rotor blades.

The bars (A and B) between points 1 and 2, and 1 and 3, may be interpreted as representing the structure at the hub while the bar (C) between points 2 and 3 may be regarded as representing the fuselage structure. These three bars can be treated dynamically as springs with elastic moduli k_A, k_B ($=k_A$) and k_C, respectively, where k_A represents the elastic resistance of the hub and k_C that of the fuselage*. If the bars are regarded as ordinary structural components of a framework, then, by Hooke's law,

$$k_A = E_A S_A / l_A \quad , \quad k_C = E_C S_C / l_C \qquad (1)$$

It will be seen subsequently that it is more convenient to replace k_A and k_C by parameters proportional to the natural frequencies of the hub support and of the fuselage.

---

*k has the usual meaning of force per elongation.

It is assumed that given external harmonic forces of the form

$$F_z = F_1 \cos \nu \Omega t \qquad (2a)$$

$$F_x = F_2 \cos \nu \Omega t \qquad (2b)$$

act at point 1 (Figure 1) in the z and x directions, respectively, where z is normal to the rotor plane of rotation, and x is perpendicular to z and in the plane of the (simplified) structure of hub and fuselage. F_1 and F_2 are given constants, i.e. independent of time, and are considered as representing the magnitudes of the net forces transmitted by the rotor blade system to the hub in directions normal to, and in, the plane of rotation, respectively. If the blades are balanced, and only a single harmonic is assumed to be transmitted to the hub, then, as shown in e.g., reference 1,

$$\nu = n \qquad (3)$$

where n is the number of rotor blades.

II

Basic Dynamic Equations

The equations of motion for each mass point in Figure 1 can be derived by applying D'Alembert's theorem of inertia forces, and assuming that each bar exerts a restoring force which acts in the direction of the longitudinal axis of the bar, and which has a magnitude proportional to the elongation and to the spring modulus (k) of the bar. Let u_i and v_i denote the displacements of the mass point i in the x and z directions, respectively, and let it be assumed that these displacements are sufficiently small so that only first-degree terms in these displacements need be taken into account. Then the elongation (Δl) of each bar will be:

$$\begin{aligned}
(\Delta l)_A &= (u_1-u_2) \cos \theta + (v_1-v_2) \sin \theta \\
(\Delta l)_B &= (u_3-u_1) \cos \theta + (v_1-v_3) \sin \theta \\
(\Delta l)_C &= u_3-u_2
\end{aligned} \qquad (4)$$

The conditions of equilibrium of inertia, elastic and external forces for each of the three mass points in the x and z directions yield, with the use of equations (2a), (2b) and (4), the following six linear differential equations in the six unknowns $u_i(t)$, $v_i(t)$ $(i=1, 2, 3)$:

$$-m_1\ddot{u}_1+k_A[(u_2-2u_1+u_3)\cos\theta+(v_2-v_3)\sin\theta]\cos\theta+F_2\cos\nu\Omega t=0 \tag{5a}$$

$$-m_1\ddot{v}_1-k_A[(u_3-u_2)\cos\theta+(2v_1-v_2-v_3)\sin\theta]\sin\theta+F_1\cos\nu\Omega t=0 \tag{5b}$$

$$-m_2\ddot{u}_2+k_A[(u_1-u_2)\cos\theta+(v_1-v_2)\sin\theta]\cos\theta+k_C(u_3-u_2)=0 \tag{5c}$$

$$-m_2\ddot{v}_2+k_A[(u_1-u_2)\cos\theta+(v_1-v_2)\sin\theta]\sin\theta=0 \tag{5d}$$

$$-m_2\ddot{u}_3+k_A[(u_1-u_3)\cos\theta+(v_3-v_1)\sin\theta]\cos\theta+k_C(u_2-u_3)=0 \tag{5e}$$

$$-m_2\ddot{v}_3+k_A[(u_3-u_1)\cos\theta+(v_1-v_3)\sin\theta]=0 \tag{5f}$$

As indicated previously, the hub and fuselage structural properties can be characterized by natural frequencies instead of by the spring moduli, although the natural frequencies are determined by the spring moduli. Such natural frequencies can be obtained by first considering the motion of the hub without the motion of the fuselage. In the simplified structure of Figure 1, this can be interpreted as letting points 2 and 3 be fixed, while only point 1 is free to move. The natural frequency thus obtained would be that of a structure composed of (massless) springs A and B with ends fixed in a fixed rigid bar C, and with a mass m_1 at point 1.

An expression for this natural frequency can be derived from equations (5a) and (5b) by setting $F_1=F_2=u_2=u_3=v_2=v_3=0$, and noting that the solutions of the two resulting differential equations denote simple harmonic motions with frequencies ω_{hu} and ω_{hv}, respectively, given by:

$$\omega_{hu}^2 = \frac{2k_A\cos^2\theta}{m_1} \tag{6a}$$

$$\omega_{hv}^2 = \frac{2k_A\sin^2\theta}{m_1} \tag{6b}$$

These frequencies can be interpreted as the natural frequencies of the hub structure for motion in, and normal to, the plane of rotation, respectively. ω_{hu} and ω_{hv} can, if desired, also be interpreted simply as reference natural frequencies <u>defined</u> by equations (6a) and (6b). For $\theta=45^{\circ}$, which is the case to be worked out in detail in the present analysis, $\omega_{hu} = \omega_{hv} = \omega_h$ (say), where

$$\omega_h^2 = \frac{k_A}{m_1} \tag{7}$$

An expression for a natural frequency denoting the elastic stiffness of the fuselage (without hub) can be obtained in an analogous fashion. Here, one can consider the motion, in the x direction, of the structure composed of the two masses m_2 connected by the (massless) spring of modulus k_C. The natural frequency of this system can be derived by setting $k_A = 0$ (i.e. fuselage isolated from hub) in equations (5c) and (5e), and noting that the simple harmonic solution of these equations then has a frequency ω_f given by:

$$\omega_f^2 = \frac{2\ k_C}{m_2} \tag{8}$$

Thus, k_C can be replaced by the physically more significant parameter ω_f, which again may be regarded as either a reference frequency <u>defined</u> by equation (8), or as the natural frequency of the fuselage structure, unaffected by the hub or pylon connections.

III
General Solution of Equations

To obtain the general solution of equations (5a)-(5f), no significant loss in generality will be incurred if θ is given a specific value. Consequently, it will now be assumed that $\theta=45^{0}$. Moreover, it will be convenient to give the solution separately for $F_1=0$, and for $F_2=0$, since the solution for $F_1 \neq 0$, and $F_2 \neq 0$ simultaneously will simply be the sum of these two separate solutions.

Equations (5a)-(5f) can be solved by setting

$$\left.\begin{aligned} u_i &= e_{ixt} \cos \nu\Omega t \\ v_i &= e_{izt} \cos \nu\Omega t \\ &(i = 1, 2, 3) \end{aligned}\right\} \qquad (9)$$

where e_{ixt} and e_{izt} are constants. Six linear algebraic equations in the six unknowns e_{ixt} and e_{izt} are thus obtained. For $\underline{\theta=45^\circ}$ and $\underline{F_1=0}$, the solution of these equations is found to be:

$$e_{1xt} = \frac{F_2}{m_1 \Omega^2} \frac{1-(MH^2/\nu^2)}{(1+M)H^2 - \nu^2} \qquad (10a)$$

$$e_{2xt} = -\frac{F_2}{2m_1 \Omega^2} \frac{(MH^2/\nu^2)}{(1+M)H^2 - \nu^2} \qquad (10b)$$

$$e_{3xt} = e_{2xt} \qquad (10c)$$

$$e_{1zt} = 0 \qquad (10d)$$

$$e_{2zt} = e_{2xt} \qquad (10e)$$

$$e_{3zt} = -e_{2xt} \qquad (10f)$$

For $\underline{\theta = 45^\circ}$ and $\underline{F_2 = 0}$, the solution is found to be (using primes to denote the case $F_2 = 0$):

$$e_{1xt}' = 0 \qquad (11a)$$

$$e_{2xt}' = -\frac{F_1}{2m_1 \Omega^2} \frac{M}{M(\nu^2 - \frac{\varphi^2}{2}) + (1 - \frac{\nu^2}{H^2})(\nu^2 - \varphi^2)} \qquad (11b)$$

$$e_{3xt}' = -e_{2xt}' \tag{11c}$$

$$e_{1zt}' = \frac{F_1}{m_1\Omega^2}\;\frac{-M\left(1-\frac{\varphi^2}{2\nu^2}\right)+\frac{\nu^2}{H^2}\left(1-\frac{\varphi^2}{H^2}\right)}{M\left(\nu^2-\frac{\varphi^2}{2}\right)+\left(1-\frac{\nu^2}{H^2}\right)\left(\nu^2-\varphi^2\right)} \tag{11d}$$

$$e_{2zt}' = -\frac{F_1}{2m_1\Omega^2}\;\frac{M\left(1-\frac{\varphi^2}{\nu^2}\right)}{M\left(\nu^2-\frac{\varphi^2}{2}\right)+\left(1-\frac{\nu^2}{H^2}\right)\left(\nu^2-\varphi^2\right)} \tag{11e}$$

$$e_{3zt}' = e_{2zt}' \tag{11f}$$

where the non-dimensional parameters are defined as follows:

$$\left.\begin{aligned} M &\equiv m_1/m_2 \\ H &\equiv \omega_h/\Omega \\ \varphi &\equiv \omega_f/\Omega \end{aligned}\right\} \tag{12}$$

M is a mass-ratio parameter, while H and φ are elastic parameters of the hub and the fuselage structure, respectively.

Since the structure analyzed here is suspended freely in space, it follows that the three mass points can be displaced in such a manner that the entire structure moves as a rigid body. The expressions given in equations (10a)-(10f) and (11a)-(11f), therefore, include rigid-body displacements, and denote the sum of such displacements and elastic displacements. (The subscript "t" has consequently been used, to denote "total" displacements.) The rigid-body displacements themselves can be derived from equations (10a)-(11f) by letting $H \longrightarrow \infty$ and $\varphi \longrightarrow \infty$ there, i.e. by letting the hub and fuselage become infinitely stiff. The following expressions (denoted by subscript "r") are thus obtained for the rigid-body displacements (with $\theta = 45^{\circ}$) for $F_1 = 0$ and for $F_2 = 0$, respectively:

$$e_{1xr} = -\left(\frac{F_2}{m_1 \Omega^2 \nu^2}\right)\left(\frac{M}{1+M}\right) \tag{13a}$$

$$e_{2xr} = e_{1xr}/2 \tag{13b}$$

$$e_{3xr} = e_{1xr}/2 \tag{13c}$$

$$e_{1zr} = 0 \tag{13d}$$

$$e_{2zr} = e_{1xr}/2 \tag{13e}$$

$$e_{3zr} = -\, e_{1xr}/2 \tag{13f}$$

$$e_{1xr}' = 0 \tag{14a}$$

$$e_{2xr}' = 0 \tag{14b}$$

$$e_{3xr}' = 0 \tag{14c}$$

$$e_{1zr}' = -\left(\frac{F_1}{m_1 \Omega^2 \nu^2}\right)\left(\frac{M}{2+M}\right) \tag{14d}$$

$$e_{2zr}' = e_{1zr}' \tag{14e}$$

$$e_{3zr}' = e_{1zr}' \tag{14f}$$

As a check on these expressions it can be verified from equations (4) that if the displacement amplitudes are given by equations (13a)-(14f), then the elongation of each bar will be zero.

The displacements which are probably of the most pratical interest here are the elastic displacements, which may be denoted as e_{1x} and e_{1z} (subscripts "r" and "t" are omitted), and which can be defined by relations of the form:

$$e_{1x} = e_{1xt} - e_{1xr}, \text{ etc.} \tag{15}$$

Substitution of equations (10), (11), (13) and (14) into equations (15) leads to the following expressions for the elastic deflections of the three mass points:

$$e_{1x} = \frac{F_2}{m_1 \Omega^2} \cdot \frac{1}{1 + M} \cdot \frac{1}{(1+M)H^2 - \nu^2} \tag{16a}$$

$$e_{2x} = -(M/2)\, e_{1x} \tag{16b}$$

$$e_{3x} = -(M/2)\, e_{1x} \tag{16c}$$

$$e_{1z} = 0 \tag{16d}$$

$$e_{2z} = -(M/2)\, e_{1x} \tag{16e}$$

$$e_{3z} = (M/2)\, e_{1x} \tag{16f}$$

$$e_{1x}' = 0 \tag{17a}$$

$$e_{2x}' = -\frac{F_1}{2m_1 \Omega^2} \; \frac{M}{M(\nu^2 - \frac{\varphi^2}{2}) + (1 - \frac{\nu^2}{H^2})(\nu^2 - \varphi^2)} \tag{17b}$$

$$e_{3x}' = -e_{2x}' \tag{17c}$$

$$e_{1z}' = \frac{F_1}{m_1 \Omega^2} \; \frac{1}{2+M} \; \frac{(2/H^2)(\nu^2 - \varphi^2) - M}{M(\nu^2 - \frac{\varphi^2}{2}) + (1 - \frac{\nu^2}{H^2})(\nu^2 - \varphi^2)} \tag{17d}$$

$$e_{2z}' = -(M/2) e_{1z}' \tag{17e}$$

$$e_{3z}' = -(M/2) e_{1z}' \tag{17f}$$

If more than a single external harmonic force acts at the hub, for example if $F_x = \sum_{\nu} F_{2\nu} \cos \nu \Omega t$, then equations (16a)-(17f) remain valid for each ν, with F_2, for example, replaced by $F_{2\nu}$, and the resulting expressions for each ν need merely be added. In the subsequent discussion, however, it will be assumed that only a single exciting harmonic force acts at the hub, and that ν can be replaced by n, the number of rotor blades (cf. equation (3)).

IV

Resonance and Other Implications of General Solution

Equations (16a)-(17f) are particularly simple general expressions, quite convenient for calculation, giving the response of the hub-fuselage structure to harmonic forces transmitted by the blades to the hub in flight. The terms with subscript "x" denote vibrational motions in the rotor plane of rotation, while those with subscript "z" denote motions normal to the plane.

It should be noted that for $F_1=0$, i.e. for external harmonic forces acting only in the plane of rotation, the deflections, as functions of the pertinent parameters, will be of the form:

$$e = \frac{F_2}{m_1 \Omega^2} \cdot \psi(M, H) \tag{18a}$$

For $F_2=0$, i.e. for external harmonic forces acting only normal to the plane of rotation, the deflections will be of the form:

$$e' = \frac{F_1}{m_1 \Omega^2} \cdot \psi'(M, H, \varphi) \tag{18b}$$

Thus, the deflections are proportional to the parameters $(F_2/m_1\Omega^2)$ or $(F_1/m_1\Omega^2)$, and are functions of the three dimensionless parameters M, H and φ. For forces acting only in the plane of rotation, however, the deflections are seen to be independent of the fuselage frequency parameter φ.

Because of the fact that in practice $M \ll 1$, it will be found that for external harmonic forces in the plane of rotation, e_{1x} will ordinarily have the largest magnitude. This means that the mass point 1 (Figure 1), corresponding to the hub-rotor blade connections, will vibrate with greater amplitude than mass points 2 and 3, corresponding to the hub-fuselage connections. This is, perhaps, to be expected, since the external harmonic forces, transmitted by the blades to the hub, are assumed to act at point 1. However, for an external harmonic force acting normal to the rotor plane of rotation, it will be seen subsequently that points 1 and 2 may vibrate with roughly equal amplitudes under certain special conditions (cf. equations (21) and (22) below).

The conditions of resonance are of particular practical importance, since they indicate the conditions which should be avoided in actual design. Resonance may be defined here as the condition under which the magnitudes of the deflections, as given by equations (16a)-(17f), become indefinitely large. For the system analyzed here, there will be two different types of resonance conditions, namely one for a force acting in the plane of rotation $(F_1=0)$, and one for a force acting normal to the plane of rotation $(F_2=0)$.

For a <u>force in the plane of rotation</u>, equations (16a)-(16f) imply that resonance will occur when

$$(1+M)\ H^2 - n^2 = 0 \tag{19a}$$

Therefore the value $(H_{res.})$ of H for resonance is

$$H_{res.} = \frac{n}{\sqrt{1+M}} \tag{19b}$$

Thus, resonance will occur for a "hub natural frequency" ω_h slightly less than $n\Omega$.

For a force normal to the plane of rotation, equations (17a)-(17f) imply the following condition for resonance:

$$M\left(n^2 - \frac{\varphi^2}{2}\right) + \left(1 - \frac{n^2}{H^2}\right)\left(n^2 - \varphi^2\right) = 0 \qquad (20a)$$

or

$$\varphi_{res}^{\;2} = n^2 \; \frac{n^2 - (1+M)\,H^2}{n^2 - \left(1+\frac{M}{2}\right)H^2} \qquad (20b)$$

Equation (20b) will ordinarily imply that for a given H, $\varphi^2 \approx n^2$ for resonance, i.e. for a given hub natural frequency parameter H, resonance will occur when the "fuselage natural frequency" ω_f is approximately equal to $n\Omega$. For a given fuselage natural frequency parameter φ, the value of H^2 for resonance due to an exciting force normal to the plane of rotation, according to equation (20a), will be:

$$H_{res}^{\;2} = n^2 \; \frac{n^2 - \varphi^2}{n^2(1+M) - \varphi^2\left(1+\frac{M}{2}\right)} \qquad (20c)$$

Thus, for a given φ, $H_{res} \approx n$, or $\omega_f \approx n\Omega$, for resonance. If $\varphi < n$, then $H_{res.} < n$, but if $n < \varphi < \sqrt{2}\,n$, then $H_{res.} > n$ (unlike the case of resonance due to an exciting force in the plane of rotation).

For a fixed ratio $f(\equiv \omega_f/\omega_h \equiv \varphi/H)$ of fuselage to hub natural frequency, the condition of resonance is somewhat different in nature from that (cf. equation (20c)) for a fixed fuselage natural frequency. For a fixed f, in fact, there will in general be two values of H for resonance, given according to equation (20a) by:

$$H_{res}^{\;2} = n^2 \; \frac{1+M+f^2 \pm \sqrt{(f^2-1)^2 + M(2+M)}}{f^2\,(2+M)} \qquad (20d)$$

Equation (20d), with M (necessarily) positive, implies that if (and only if) $0 < f < \sqrt{2}$, then one of the values of $H_{res.}$

will be less than n, while the other will exceed n. Equations (20a)-(20d) are, of course, all mathematically equivalent, but indicate the different points of view discussed above.

It may be of interest to observe the asymptotic behavior of the elastic deflections as the natural frequencies of the hub and of the fuselage become infinitely large, i.e. as the hub-fuselage structure becomes infinitely stiff. As $H \rightarrow \infty$, it follows from equations (16a)-(16f) that e_{1x} and $e_{1z} \rightarrow 0$. Moreover, as $H \rightarrow \infty$ while f is fixed, and hence $\varphi \rightarrow \infty$, equations (17a)-(17f) imply that e_{1x}' and $e_{1z}' \rightarrow 0$. These results are, of course, to be anticipated on physical grounds. However, if φ (instead of f) is fixed, then as $H \rightarrow \infty$, it follows from equations (17b) and (17d) that

$$\left.\begin{aligned} (e_{2x}')_{H\rightarrow\infty} &\rightarrow -\frac{F_1}{2m_1\Omega^2}\;\frac{M}{n^2(1+M)-\varphi^2(1+\frac{M}{2})} \\ (e_{1z}')_{H\rightarrow\infty} &\rightarrow \frac{2}{2+M}\,(e_{2x}')_{H\rightarrow\infty} \end{aligned}\right\} \quad (21)$$

Relations (21) indicate that with the fuselage natural frequency parameter φ fixed, the vibrations due to a harmonic force normal to the plane of rotation will approach a finite (i.e. non-zero) magnitude as the hub natural frequency becomes indefinitely large. This result can be explained by noting that in this case, since φ is fixed, the fuselage elastic stiffness does <u>not</u> at the same time also become indefinitely large.

It may be observed from equations (17a)-(17f) that in the special case of $\varphi = \nu\,(=n)$, i.e. $\omega_f = n\Omega$, the response of the hub-fuselage structure to a harmonic force (of frequency $n\Omega$) normal to the plane of rotation will be independent of H, i.e. independent of the hub stiffness. Hence resonance cannot occur in this special case for any non-vanishing value of H. In this case, in fact, equations (17b) and (17d) yield:

$$\left.\begin{aligned} (e_{2x}')_{\varphi=n} &= -\frac{F_1}{m_1 \Omega^2 n^2} \\ (e_{1z}')_{\varphi=n} &= -\frac{F_1}{m_1 \Omega^2 n^2}\left(\frac{2}{2+M}\right) \end{aligned}\right\} \quad (22)$$

In this case, therefore, as also in the case $H \rightarrow \infty$ with φ fixed (cf. equations (21)), point 2 (Figure 1) will have an amplitude of deflection slightly greater than that of point 1.

It is interesting, finally, to note that for an external harmonic force acting normal to the rotor plane of rotation, a condition exists under which the elastic vibrational motions of the hub-fuselage structure will occur only in the plane of rotation. From equations (17d)-(17f) it is seen that this will occur when

$$2(n^2 - \varphi^2) - MH^2 = 0$$

or

$$H^2 = \frac{2}{M}(n^2 - \varphi^2) \quad (23)$$

When equation (23) is satisfied, then since only e_{2x}' and e_{3x}' will not vanish (cf. equations (17a)-(17f)), the forced elastic vibrations of the hub-fuselage due to a harmonic force (of frequency $n\Omega$) normal to the plane of rotation will consist only of vibrations of points 2 and 3 (fuselage-hub connections, Figure 1) in the plane of rotation.

V

Numerical Examples

To illustrate more explicitly the results of the present analysis, numerical examples will be worked out, based on the following data, which is fairly typical:

$n = 3$ (three-bladed rotor)

$m_1 = 9.30$ slugs (=300 lbs.)

$\Omega = 28$ rad./sec. (=268 r. p. m.)

$M = 0.15$

It will be assumed, moreover, that

$F_1 = 400$ lbs., $\qquad F_2 = 1000$ lbs.

The values for F_1 and F_2 used here are based on the theoretical results of the numerical example carried out in reference 1 for the forces transmitted by the blades to the hub in flight. [For a three-bladed helicopter of gross weight 4660 lbs., it was found, in fact, that a net harmonic force of magnitude 400 lbs. (and frequency $3\,\Omega$) would be transmitted in the direction normal to the plane of rotation, while a net harmonic load of magnitude 1454 lbs. would be transmitted in the plane of rotation.] Since the amplitudes of the vibrations are directly proportional to F_1 and F_2, it follows that for any values of F_1 and F_2 other than those assumed in the present examples, the deflections can be readily obtained from those to be given here.

Since the largest deflections (in absolute value) in these examples will be $|e_{1x}|$ due to the force (given by F_2) in the plane of rotation, and $|e_{1z}'|$ due to the force (given by F_1) normal to the plane of rotation, only these two quantities need be considered here in detail.

From equation (16a), it follows, with the present data, that

$$e_{1x} = \frac{1.240}{H^2 - 7.81} \text{ inches} \qquad (24)$$

Equation (24) for $|e_{1x}|$ vs. H is plotted in Figure 2. It is interesting to compare these results with those of reference 4, which are based on a different type of analysis. The case in reference 4 corresponding most closely to the plane structure treated here is probably $\Omega_x / \Omega_y = 0$ (in the notation of reference 4), i.e. infinite stiffness in a direction normal to the plane of the structure. Equation (24) is seen to have the same form as the expression for hub response in Figure 4 of reference 4, and the resonance condition based on equation (24), namely $H_{res} = 2.79$, is seen to be almost equivalent to that, namely $H_{res} = 2.72$, of reference 4. The amplitudes of deflection are considerably greater here than in reference 4, but this is due to the fact that the numerical results here are based on a much larger external harmonic force* ($F_2 = 1000$ lbs.) than that assumed in reference 4 (where a load of 50 lbs. per blade in the plane of rotation is assumed, so that $F_2 = (3/2) \times 50 = 75$ lbs.). If the deflections in Figure 2 are multiplied by 75/1000, then the deflections will be found to be of the same order of magnitude as those in reference 4, with the present results indicating a somewhat smaller response.

Substitution of the data assumed here into equation (17d) yields:

$$e_{1z}' = \frac{0.305}{H^2} \; \frac{2(9-\varphi^2)-0.15H^2}{(10.35 - 1.075\varphi^2) - \frac{9}{H^2}(9-\varphi^2)} \text{ inches} \qquad (25a)$$

In terms of the fuselage-hub natural frequency ratio $f(=\varphi/H)$, equation (25a) can be written in the form:

$$e_{1z}' = 0.305 \; \frac{(18/H^2) - 0.15 - 2f^2}{10.35 - f^2(9-1.075H^2)-(81/H^2)} \text{ inches} \qquad (25b)$$

Equations (25a) and (25b) for $|e_{1z}'|$ as a function of H for various fixed fuselage natural frequencies (fixed φ), and as a function of H for various fixed fuselage-hub natural frequency ratios (f), are shown plotted in Figures 3 and 4, respectively. The resonance conditions are also tabulated there**.

---

* As shown explicitly in reference 1, this force is due not only to drag, but also to Coriolis, lift, inertia and centrifugal components.

** One of the choices for f in Figure 4, namely $f=\sqrt{2M}$, was made on the basis of supposing $k_C=k_A$, and comparing equations (7) and (8).

Conclusions

By using a simple model to represent the helicopter hub and fuselage, an analysis has been made of the hub-fuselage response to harmonic forces transmitted in flight by the rotor blades to the hub both in, and normal to, the plane of rotation. The expressions derived for the amplitudes of the forced vibrations both in and normal to the plane of rotation are particularly simple, and are very convenient for calculations. The resonance conditions thus derived are also quite simple. The pertinent non-dimensional parameters are: the number of blades (n), the ratio of hub natural frequency to rotor angular speed ($H=\omega_h/\Omega$), the ratio of fuselage natural frequency to rotor speed ($\varphi=\omega_f/\Omega$) and the ratio of blade and pylon mass to half of the fuselage mass ($M=m_1/m_2$). The vibration amplitudes, moreover, are proportional to ($F/m_1\ \Omega^2$), where F denotes the amplitude of the exciting harmonic force. The vibrations due to a force in the plane of rotation are independent of the fuselage natural frequency parameter (φ).

Despite the simplicity of the physical structure analyzed, and of the final analytical expressions, the results of a typical numerical example based on the present analysis agree well with those based on a considerably different, and somewhat more involved, type of analysis for the hub response in the rotor plane of rotation due to exciting harmonic forces in this plane.

As an extension of the present analysis, a space-framework type of structure, instead of the plane structure treated here, can be assumed as a physical model of the hub-fuselage. In this manner it would be possible to take into account the effect of the actual rotation (cf., for example, reference 1) of the harmonic force transmitted by the rotor blades to the hub in the plane of rotation.

References

1. M. Morduchow, S. W. Yuan and K. E. Peress, "Helicopter Blade-Forces Transmitted to the Rotor Hub in Flight", P.I.B.A.L. Report No. 22, May 1953, Submitted to Air Force for review and approval for publication.

2. M. L. Deutsch, "Theory of Mechanical Instability of Rotors", A.A.F. Report 459-1-18, January 1943.

3. R. P. Coleman, "Theory of Self-Excited Mechanical Oscillations of Hinged Rotor Blades", N.A.C.A. Wartime Report A.R.R. 3G29, July 1943.

4. O. R. Rogers and W. Oleksak, "The Response of Helicopter Rotors to Oscillatory Rotor Plane Drag Forces at the Blades", W.A.D.C. Technical Report 52-270, RDO No. 455-45, September 1952.

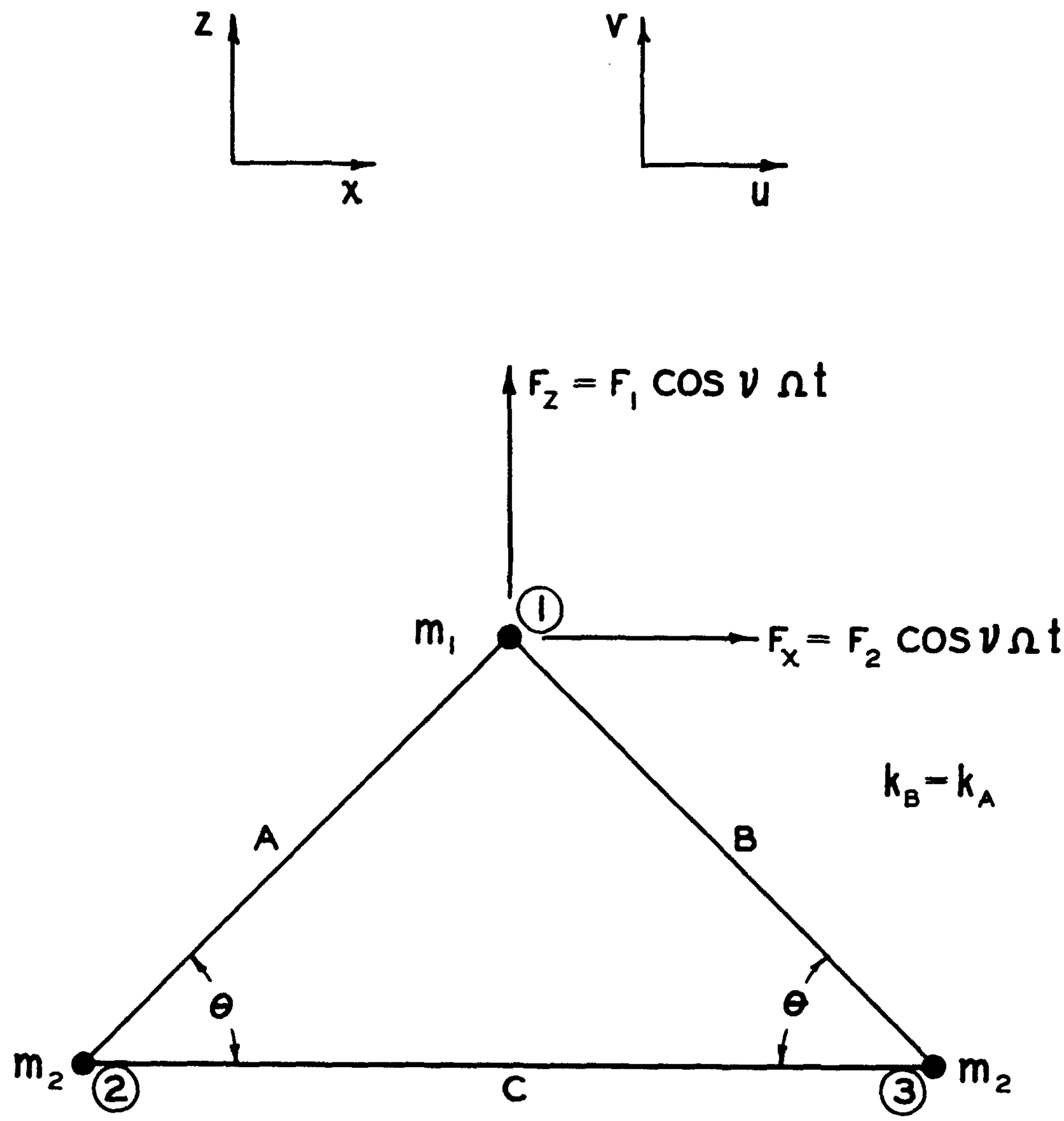

FIG. 1 SIMPLIFIED HUB-FUSELAGE STRUCTURE

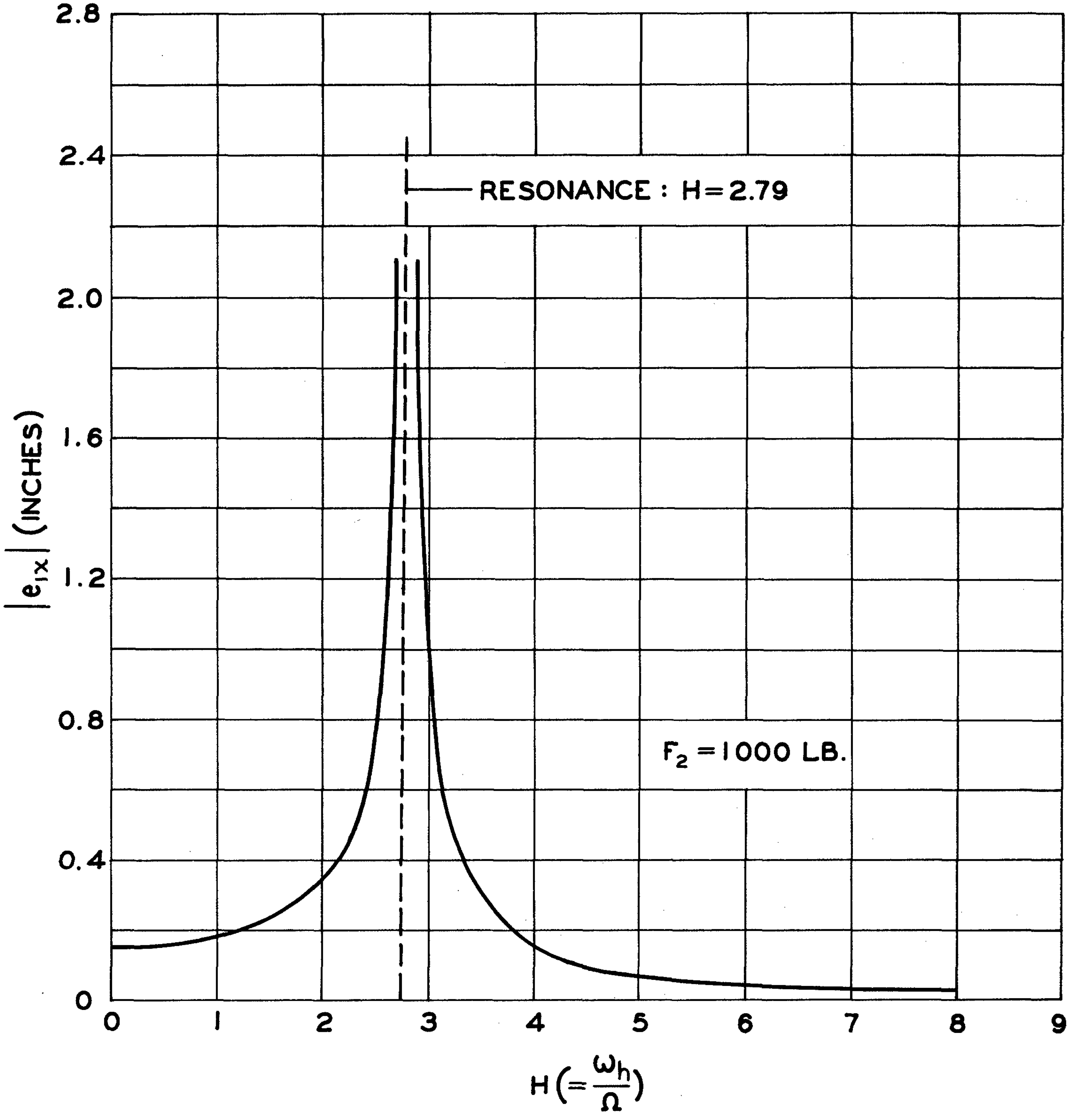

FIG. 2
RESPONSE OF HUB TO HARMONIC FORCE IN PLANE OF ROTATION

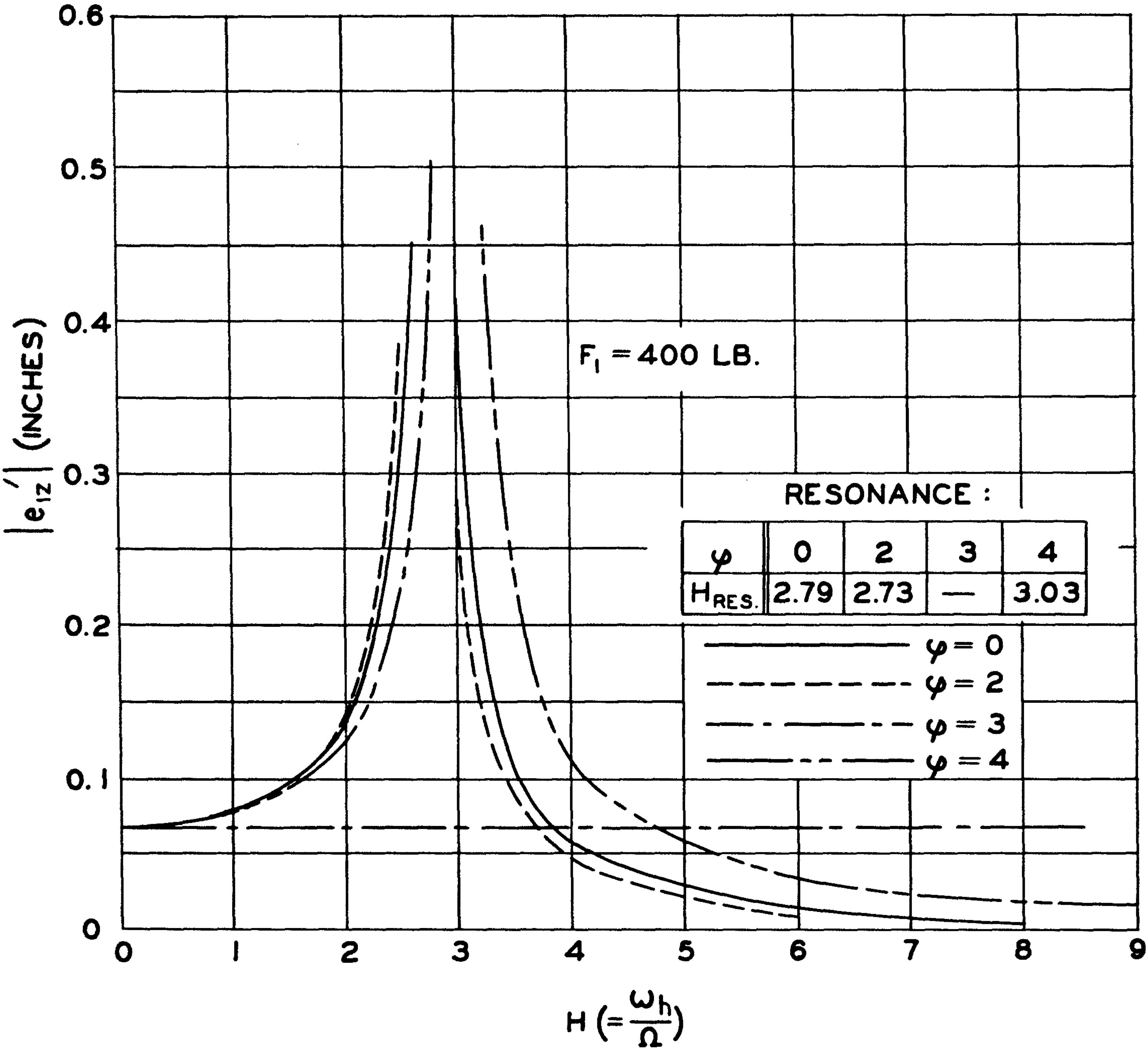

FIG. 3

RESPONSE OF HUB TO HARMONIC FORCE NORMAL TO PLANE OF ROTATION FOR FIXED NATURAL FREQUENCIES OF FUSELAGE

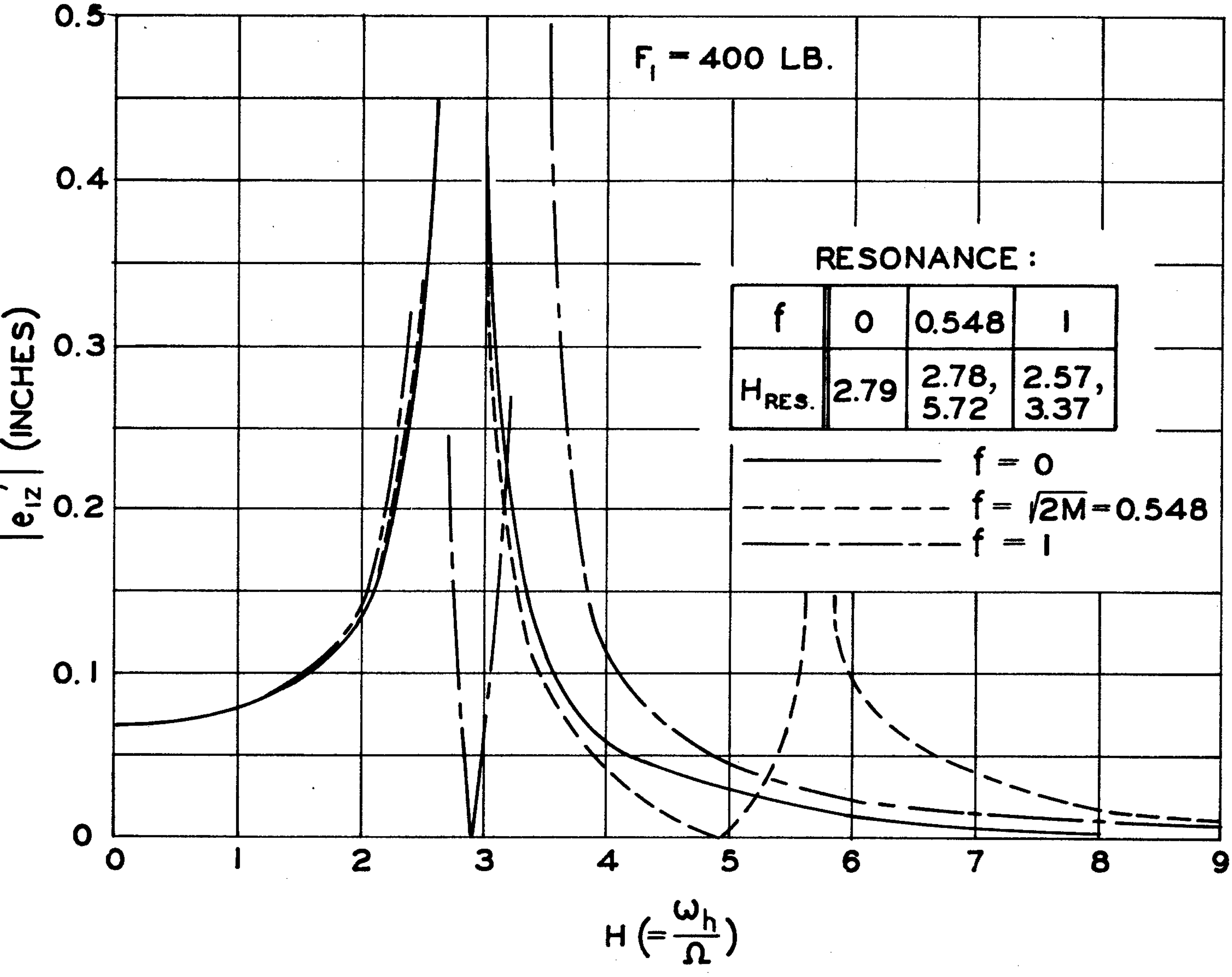

FIG. 4

RESPONSE OF HUB TO HARMONIC FORCE NORMAL TO PLANE OF ROTATION FOR FIXED RATIOS OF FUSELAGE TO HUB NATURAL FREQUENCIES